An Overview of Quadratic Gravity

John Donoghue

On Quadratic Gravity

John F. Donoghue[1*] and Gabriel Menezes[2†]
[1] *Department of Physics, University of Massachusetts, Amherst, MA 01003, USA*
[2] *Departamento de Física, Universidade Federal Rural do Rio de Janeiro, 23897-000, Seropédica, RJ, Brazil*

We provide a brief overview of what is known about Quadratic Gravity, which includes terms quadratic in the curvatures in the fundamental action. This is proposed as a renormalizeable UV completion for quantum gravity which continues to use the metric as the fundamental dynamical variable. However, there are unusual field-theoretic consequences because the propagators contain quartic momentum dependence. At the present stage of our understanding, Quadratic Gravity continues to be a viable candidate for a theory of quantum gravity.

My work is with Gabriel Menezes:
arXiv:1712.04468 , arXiv:1804.04980, arXiv:1812.03603
arXiv:1908.02416, arXiv:1908.04170, aXiv:2003.09047
arXiv:2105:00898, arXiv:2106.05912, arXiv:2112:01974 …
But this understanding builds on the past work of many others

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Connections:

Early pioneers: **Stelle**, Fradkin-Tsetlyn, Julve-Tonin Adler, Zee, Smilga,
Tomboulis, Hasslacher-Mottola,
Lee-Wick, Coleman, Boulware-Gross….

Present activity: Einhorn-Jones, Salvio-Strumia, Holdom-Ren,
Donoghue-Menezes, Mannheim, Anselmi
Odintsov-Shapiro, Narain-Anishetty…

In the neighborhood: Lu-Perkins-Pope-Stelle, ‘t Hooft,
Grinstein-O’Connell-Wise ….

Outline

1) **Overview of the overview**

2) **Ostrogradsky and stability**

3) **Causality**

4) **Unitarity**

5) **Summary/ open issues**

Quadratic gravity – the good news:

$$S_{\rm quad} = \int d^4x\sqrt{-g}\left[\frac{2}{\kappa^2}R + \frac{1}{6f_0^2}R^2 - \frac{1}{2\xi^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}\right]$$

Renormalizeable QFT for quantum gravity

- propagator modification tames the UV

$$R \sim \partial^2 g \qquad\qquad R^2 \sim \partial^2 g\partial^2 g$$

Uses metric as basic field

- only stable particle is the massless graviton

Can be **weakly coupled** at high energy

- for $\xi \ll 1$

The **most conservative** version of quantum gravity

Quadratic gravity – the bad news:

BUT:

$$R \sim \partial^2 g \qquad\qquad R^2 \sim \partial^2 g \partial^2 g$$

$$iD_2(q) = \frac{i}{q^2 - \frac{q^4}{M^2}} = \frac{i}{q^2} - \frac{i}{q^2 - M^2}$$

Higher derivative theories "break" quantum field theory

- General principles* lead to Källen-Lehman representation

$$D(q) = \frac{1}{\pi} \int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Quadratic gravity must break some fundamental ingredient of QFT

- But not general agreement as to what breaks

*There is a potential caveat – gauge current – but seems not relevant for spin 2

Caution: not all approaches need be equivalent

Usual way we teach/discuss theories:

1) Classical physics and solutions
2) Canonical Hamiltonian quantization of free field theory
3) Add interactions
4) Repeat with Lagrangian Path Integrals

Here – reverse pathway:

1) Start with Lorentzian Lagrangian Path Integral
2) Include interactions with matter (also leading self interactions)
3) Then, analyze gravitational sector
4) Limits to standard EFT at low energy (and classical physics)

Reverse pathway is like the approach to Electroweak theory

Our understanding of the equivalence is based on standard theories

Why this path?

Spectrum becomes clear at first step
- only stable state is massless graviton

No need for canonical quantization of unstable ghost

Only stable states appear in unitarity sum
- no ghosts, only their decay products

Low energy limit is usual gravitational EFT
- with usual stability properties

Also Lorentzian vs Euclidean

This equivalence is not a sacred principle

- changes in causality/spectrum
- especially problematic for gravity !

This path starts with Lorentzian PI

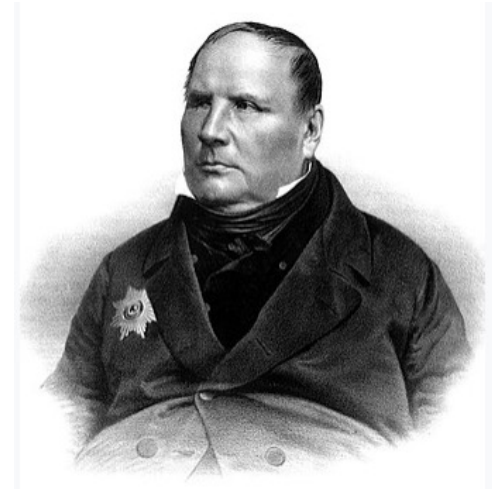
Note: Anselmi takes different path – starting from Euclidean

- perhaps the paths will merge someday

Ostrogradsky

Deep historical research from Wikipedia

- Mikhail Vasilyevich Ostrogradsky 1801- 1862
- Russian mathematician
- Educated at Sorbonne, College de France
- work on algebraic functions, calculus of variations



The Ostrogradsky instability (1850)

- theories with higher time derivatives
- requires extra canonical coordinates and canonical momenta
- Hamiltonian chosen to reproduce Hamilton's equations
- result is not positive definite – even at low energy
- **to be reviewed in more detail below**

The instability is often used to rule out higher derivative theories

Explore the physics with a very simple model

- this has the same physics as the spin 2 mode of quadratic gravity
- also the model is an entrée to the discussion of the spectrum

First "normal" version (i.e. with only two derivatives)

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_\phi - g\phi\chi^\dagger\chi$$

$$\mathcal{L}_\phi = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi\phi\right]$$

$$\mathcal{L}_\chi = \partial_\mu\chi^\dagger\partial^\mu\chi - m_\chi^2\chi^\dagger\chi - \lambda(\chi^\dagger\chi)^2$$

Think of χ as "electron" and ϕ as "photon" or "graviton"
I will use $m \ll m_\chi$ and pretend that renormalized $m \to 0$

This is then scalar analog of QED or GR

If $m = 0$ we have the classical wave equation
in long wavelength limit

Now create the dangerous version with higher derivatives

$$\mathcal{L}_\phi \to \mathcal{L}_{hd}$$

$$\mathcal{L}_{hd} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi\phi - \frac{1}{M^2}\Box\phi\Box\phi\right]$$

Here think of M as the Planck mass – beyond our present experiments

How does QFT treat this?

Is this stable at low energy? Ostrogradsky says no.

Is there a classical limit?

Is this stable at high energy?

What sacred QFT principle fails?

First: Low Energy / Classical Limit

Use path integrals to define the theory

$$Z_\phi[\chi] = \int [d\phi] e^{i\int d^4x[\mathcal{L}_{hd} - g\phi\chi^\dagger\chi]}.$$

$$\mathcal{L}_{hd} = \frac{1}{2}\left[\partial_\mu\phi\partial^\mu\phi - m^2\phi\phi - \frac{1}{M^2}\Box\phi\Box\phi\right]$$

Now use auxiliary field to remove higher derivative term (***m=0* here**)

$$Z_\phi[\chi] = \int [d\phi][d\eta] e^{i\int d^4x[\mathcal{L}(\phi,\eta)]}$$

$$\mathcal{L}(\phi,\eta) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \eta\Box\phi + \frac{1}{2}M^2\eta^2 - g\phi\chi^\dagger\chi$$

Via Gaussian integration:

$$\frac{1}{2}M^2\eta^2 - \eta\Box\phi = \frac{1}{2}M^2\left(\eta - \frac{1}{M^2}\Box\phi\right)^2 - \frac{1}{2M^2}\Box\phi\Box\phi$$

Next redefine the field variables

$$\mathcal{L}(\phi,\eta) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \eta\Box\phi + \frac{1}{2}M^2\eta^2 - g\phi\chi^\dagger\chi$$

Use $\phi(x) = a(x) - \eta(x)$

$$\begin{aligned}\mathcal{L}(a,\eta) &= \left[\frac{1}{2}\partial_\mu a\partial^\mu a - ga\chi^\dagger\chi\right] \\ &- \left[\frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]\end{aligned}$$

This totally decouples the fields

$$\begin{aligned}Z_\phi[\chi] &= \int[da]e^{i\int d^4x\left[\frac{1}{2}\partial_\mu a\partial^\mu a - ga\chi^\dagger\chi\right]} \\ &\times \int[d\eta]e^{-i\int d^4x\left[\frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]} \\ &= Z_a \times Z_\eta\end{aligned}$$

Here Z_a is just normal PI, and Z_η is **complex conjugate** of normal PI

We can do the η path integral, as a usual Gaussian integral

Add $-\epsilon \int d^4x\, \phi^2$ for convergence, then

$$\eta'(x) = \eta(x) - \int d^4x iD_{-F}(x-y)\ \chi^\dagger(y)\chi(y)$$

With $iD_{-F}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - M^2 - i\epsilon}$

This result in:

$$Z_\eta = N e^{\int d^4x d^4y \frac{1}{2} g\chi^\dagger(x)\chi(x)\ iD_{-F}(x-y)\ g\chi^\dagger(y)\chi(y)}$$

At low energy, this becomes a contact interaction

$$Z_\eta = N e^{i\int d^4x \frac{g^2}{2M^2}[\chi^\dagger(x)\chi(x)]^2}$$

The result is just a shift in λ in the χ interaction

$$\lambda \to \lambda' = \lambda - \frac{g^2}{2M^2}$$

Low energy limit is a normal theory

The original normal theory with a shifted value of λ

No sign of Ostrogradsky instability

This has a normal classical limit

Note: really not $\hbar \to 0$ because $\hbar$ is a constant.
- Classical limit is kinematics where $\hbar$ is not important
- here low energy compared to “electron” Compton wavelength

This is already sufficient evidence to refute the Ostrogradsky conclusion

What about high energy?

- there is certainly something “funny” there
- high mass pole in the propagator with the wrong signature

$$iD_F \sim \frac{i}{q^2 - \frac{q^4}{M^2}} = \frac{i}{q^2} - \frac{i}{q^2 - M^2}$$

Here is an extremely important effect – **decay to light states**

$$iD(q^2) = \frac{i}{q^2 - m^2 + i\epsilon - \frac{q^4}{M^2} + \Sigma(q)} \quad .$$

The imaginary part is fixed by a normal vacuum polarization calculation

$$\Sigma_f(q) = -\frac{g^2}{32\pi^2}\int_0^1 dx \log\left[\frac{m_\chi^2 - x(1-x)(q^2+i\epsilon)}{m_\chi^2}\right]$$

At high q^2 this has

$$\mathrm{Im}\Sigma \sim \frac{g^2}{32\pi} \equiv \gamma$$

which is positive

Positive energy

To produce or detect massive state use $\bar{\chi}\chi \to M \to \bar{\chi}\chi$

$$\mathcal{M} = g^2 D(q) \sim \frac{-g^2}{q^2 - \bar{M}^2 - i\gamma}$$

Squared matrix element is same as usual BW

Incoming and outgoing particles carry positive energy
- implies that this is a positive energy resonance

Result is positive energy resonance
(we will later see that it propagates backwards in time!)

What would Ostrogradsky say?

Extra canonical coordinates and momenta

$$
\begin{aligned}
\phi_1 &= \phi \\
\phi_2 &= \dot{\phi}
\end{aligned}
\qquad
\begin{aligned}
\pi_1 &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}} = \left(\frac{\Box + M^2}{M^2}\right)\dot{\phi} \\
\pi_2 &= \frac{\partial \mathcal{L}}{\partial \ddot{\phi}} = -\frac{\Box}{M^2}\phi \ .
\end{aligned}
$$

Hamiltonian

$$
\mathcal{H}(\phi_1, \phi_2, \pi_1, \pi_2) = \pi_1 \dot{\phi}_1 + \pi_2 \dot{\phi}_2 - \mathcal{L}
$$

But have to eliminate $\ddot{\phi}$ in favor of the coordinates and momenta

$$
\ddot{\phi} = \nabla^2 \phi - M^2 \pi_2
$$

This leads to the final Hamiltonian

$$
\mathcal{H} = \pi_1 \phi_2 + \pi_2 \left(\nabla^2 \phi - M^2 \pi_2\right) - \mathcal{L}(\phi_1, \phi_2, \nabla^2 \phi - M^2 \pi_2)
$$

The first term is the Ostrogradsky instability

- π_1 and ϕ_2 can have either sign
- this is the only place where π_1 enters the Hamiltonian

Why these choices?

Chosen to reproduce Hamilton's equations

$$\dot{\phi}_1 = \frac{\partial \mathcal{H}}{\partial \pi_1}$$
$$\dot{\phi}_2 = \frac{\partial \mathcal{H}}{\partial \pi_2}$$

The Euler Lagrange equation follows from

$$\dot{\pi}_1 = -\frac{\partial \mathcal{H}}{\partial \phi_1}$$

But from QFT point of view:

Ostrogradsky construction is not the classical limit of the QFT
- QFT appears stable

Canonical quantization also does not follow Ostrogradsky path

-Indefinite metric quantization (Lee-Wick and others)

$$\begin{aligned}\mathcal{L}(a,\eta) &= \left[\frac{1}{2}\partial_\mu a\partial^\mu a - ga\chi^\dagger\chi\right] \\ &- \left[\frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}M^2\eta^2 - g\eta\chi^\dagger\chi\right]\end{aligned}$$

Roughly:

$$\pi_\eta = \frac{\partial\mathcal{L}}{\partial\dot{\eta}} = -\dot{\eta}$$

$$[\eta(x,t),\pi_\eta(x',t)] = i\hbar\delta^3(x-x') \quad \rightarrow \quad [\eta(x,t),\dot{\eta}(x',t)] = -i\hbar\delta^3(x-x')$$

Solved by:

$$[a(p),a^\dagger(p')] = -\delta^3(p-p')$$

such that

$$H_\eta = -\int\frac{d^3p}{(2\pi)^3}\sqrt{p^2+M^2}\;a^\dagger(p)a(p)$$

yields positive energy when acting on states (!)

But I still prefer PI quantization for such theories……

Spectrum

$$S_{\text{quad}} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R + \frac{1}{6 f_0^2} R^2 - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

- The graviton propagator gets modified by q^4 terms, roughly

$$iD(q) = \frac{i}{q^2 - q^4/M^2} = \frac{i}{q^2} - \frac{i}{q^2 - M^2}$$

- Spin zero portion leads to either a tachyon or a normal resonance
  - depends on sign of f_0^2
- Spin two portion leads to either a tachyon or an **unstable ghost**
  - depends on sign of ξ^2

- Choose signs to **avoid tachyons** (i.e no poles at spacelike momenta)

Need to focus on spin-two propagator and find spectrum

The spin-two propagator (including self-energy)

$$D_{\mu\nu\alpha\beta}(q^2) = \mathcal{P}^{(2)}_{\mu\nu\alpha\beta} D(q^2)$$

$$D^{-1}(q^2) = q^2 + i\epsilon - \frac{\kappa^2 q^4}{2\xi^2(\mu)} - \frac{N_{\rm eff}}{640\pi^2}\kappa^2 q^4 \ln\left(\frac{-q^2 - i\epsilon}{\mu^2}\right)$$

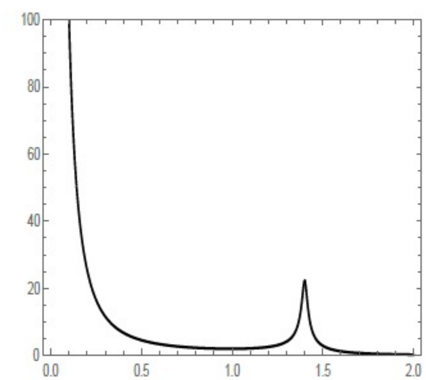
General structure:

$$iD(q) = \frac{i}{q^2 + i\epsilon - \frac{q^4}{M^2} + \Sigma(q)} \quad . \qquad \text{with} \;\; \mathrm{Im}\, \Sigma(q) = \gamma(q) \qquad \gamma(q) \geq 0$$

Massless pole is usual graviton

The high mass pole carries **two** minus sign differences:

$$\begin{aligned} iD_F(q) &= \frac{i}{q^2 - \frac{q^4}{M^2} + i\gamma(q)} \\ &= \frac{i}{\frac{q^2}{M^2}[M^2 - q^2 + i\gamma(q)(M^2/q^2)]} \\ &\sim \frac{-i}{q^2 - M^2 - i\gamma_M} \quad . \end{aligned}$$



Interpretation:

This is different from normal resonance

$$iD_F \sim_{q^2\sim m^2} = \frac{i}{q^2 - m^2 + i\gamma_m} \sim \frac{i}{q^2 - \left(m_r - i\frac{1}{2}\Gamma\right)^2}$$

Here we have

$$\sim \frac{-i}{q^2 - M^2 - i\gamma_M}$$

This is **time-reversed** version of a resonance propagator

- time reversal is anti-unitary

Still corresponds to decaying particle

Important for unitarity – **imaginary parts are the same**

$$iD(q) \sim \frac{Zi}{q^2 - m^2 + iZ\gamma} \qquad \mathrm{Im}[D(q)] \sim \frac{-\gamma}{(q^2 - m^2)^2 + \gamma^2}$$

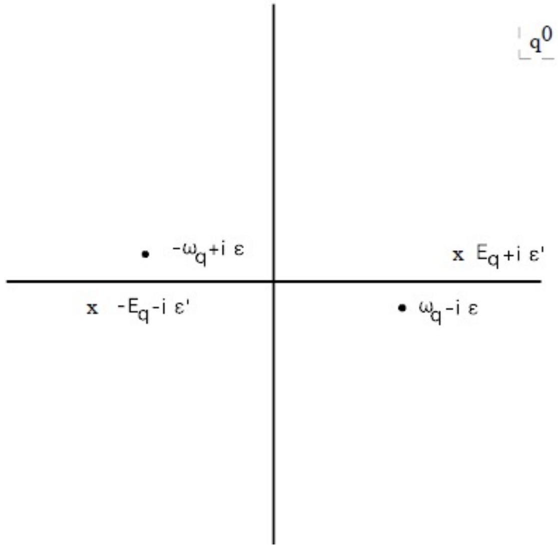
Propagator – time orderings

$$iD_F(x) = D_F^{\text{for}}(x)\theta(t) + D_F^{\text{back}}(x)\theta(-t)$$

Note **energy flow**, and also decay lifetime

$$D^{\text{for}}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{-i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})}e^{-\frac{\gamma t}{2E_q}}\right]$$

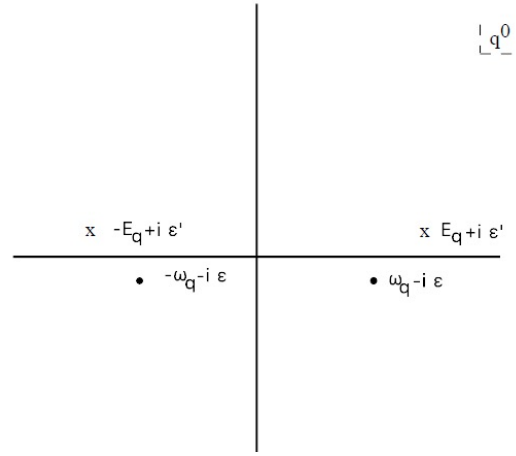
$$D^{\text{back}}(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\left[\frac{e^{i(\omega_q t - \vec{q}\cdot\vec{x})}}{2\omega_q} - \frac{e^{-i(E_q t - \vec{q}\cdot\vec{x})}}{2(E_q + i\frac{\gamma}{2E_q})}e^{-\frac{\gamma |t|}{2E_q}}\right]$$



<u>Stability in propagators:</u>

See also Salvio;
Reis, Chapiro, Shapiro

Consider propagator with retarded BC:



$$\log\left(-\left[(q_0+i\epsilon)^2-\vec{q}^2\right]\right)=\log\left(-q^2-i\epsilon q_0\right)=\log|q^2|-i\pi\theta(q^2)\left(\theta(q_0)-\theta(-q_0)\right)$$

Again propagation in both directions:

$$D_{\rm ret}(t>0,\vec{x})=D^{(0)}_{\rm ret}(t>0,\vec{x})$$

$$D_{\rm ret}(t<0,\vec{x})\equiv D^{<}_{\rm ret}(t,\vec{x})=i\int\frac{d^3q}{(2\pi)^3}\left[\frac{e^{-i(E_qt-\vec{q}\cdot\vec{x})}}{2(E_q+i\frac{\gamma}{2E_q})}e^{-\frac{\gamma|t|}{2E_q}}-\frac{e^{i(E_qt-\vec{q}\cdot\vec{x})}}{2(E_q-i\frac{\gamma}{2E_q})}e^{-\frac{\gamma|t|}{2E_q}}\right]$$

Backwards perturbations have finite lifetime:

$$h_{\mu\nu}(t,x)=\int d^3x'\left[\int_{-\infty}^{t}dt'D^{(0)}_{\rm ret}(t-t',x-x')+\int_t^\infty dt'D^{<}_{\rm ret}(t-t',x-x')\right]J_{\mu\nu}(t',x')$$

No growing modes – again no sign of Ostrogradsky instability

Causality

Known since Lee-Wick and Coleman that such propagators
lead to micro-causality violation

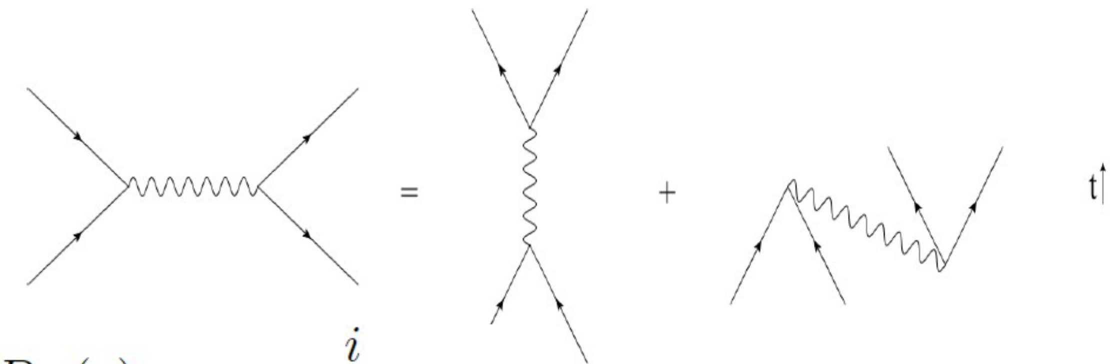
Traced to backwards-in-time propagation
- "Merlin modes"
- dueling **arrows of causality**

But limited to time scales proportional to lifetime

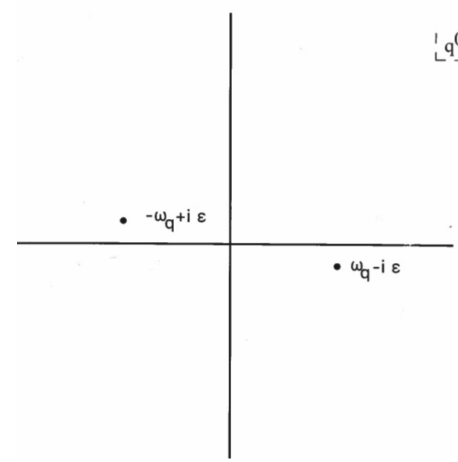
For gravity this is inverse Planck scale

This is where higher derivatives break QFT

Causality is not really “cause before effect”



$$iD_F(q) = \frac{i}{q^2 - m^2 + i\epsilon}$$



Decompose into time orderings:

$$iD_F(x) = D_F^{\text{for}}(x)\theta(t) + D_F^{\text{back}}(x)\theta(-t)$$

Positive energies propagate forward in time
- backwards propagation is “negative energy”

But backward-in-time propagation shielded by uncertainty principle
$\Delta t \sim 1/\Delta E$

Operators commute for spacelike separation

$$[\mathcal{O}(x), \mathcal{O}(x')] = 0 \quad \text{for} \quad (x - x')^2 < 0$$

Note: metric is (+,-,-,-)

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Use of Causality Conditions in Quantum Theory

M. GELL-MANN, *Institute of Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois*
M. L. GOLDBERGER,* *Princeton University, Princeton, New Jersey*

AND

W. E. THIRRING,† *Institute for Advanced Study, Princeton, New Jersey*
(Received May 24, 1954)

The limitations on scattering amplitudes imposed by causality requirements are deduced from the demand that the commutator of field operators vanish if the operators are taken at points with space-like separations. The problems of the scattering of spin-zero particles by a force center and the scattering of photons by a quantized matter field are discussed. The causality requirements lead in a natural way to the well-known dispersion relation of Kramers and Kronig. A new sum rule for the nuclear photoeffect is derived and the scattering of photons by nucleons is discussed.

This requires negative energy part of propagator to accomplish

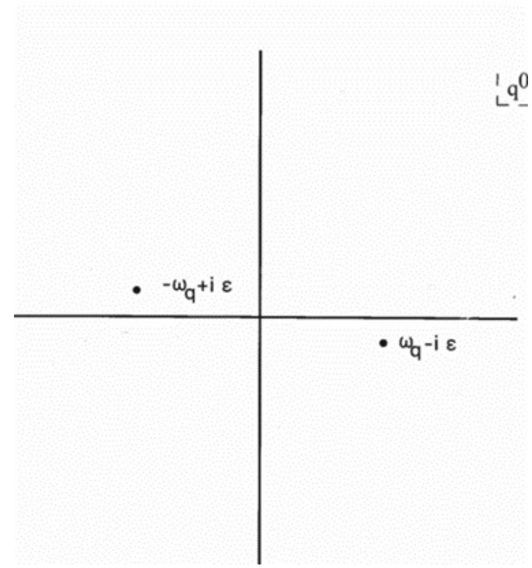
But also – Arrow of Causality

What determines past lightcone and future lightcone?
- and why do all particles share this?

This comes from the $i\epsilon$

But also – Arrow of Causality
What determines past lightcone and future lightcone?
– and why do all particles share this?
This comes from the $i\epsilon$
Determines that positive energy propagates
forward in time

Determines that positive energy propagates
forward in time



What if we used e^{-iS} instead of e^{iS}?

Consider generating functions:

$$\begin{aligned} Z_\pm[J] &= \int [d\phi] e^{\pm i S(\phi, J)} \\ &= \int [d\phi] e^{\pm i \int d^4x [\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) + J\phi]} \end{aligned}$$

Need to make this better defined – add

$$e^{-\epsilon \int d^4 x \phi^2/2}$$

Solved by completing the square:

$$Z_\pm[J] = Z[0] \exp\left\{-\frac{1}{2}\int d^4x d^4y J(x)\; iD_{\pm F}(x-y) J(y)\right\}$$

Yield propagator with specific analyticity structure

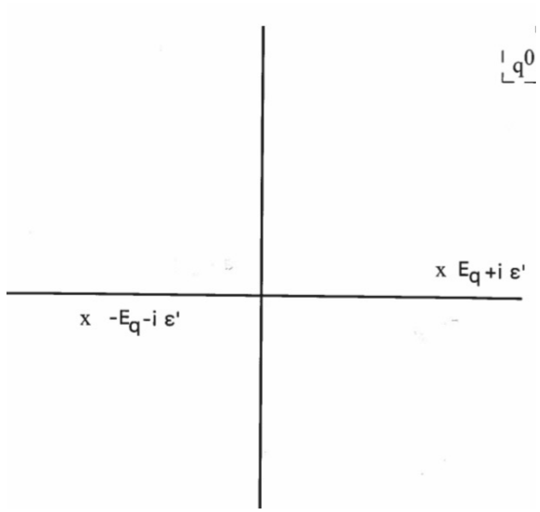
$$iD_{\pm F}(x-y) = \int \frac{d^4 q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{\pm i}{q^2 - m^2 \pm i\epsilon}$$

Result is time-reversed propagator

$$iD_{-F}(x) = D^{\rm for}_{-F}(x)\theta(t) + D^{\rm back}_{-F}(x)\theta(-t)$$

"Positive energy" propagates backwards in time

$$D^{\rm back}_{-F}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} e^{-i(E_q t - \vec{q}\cdot\vec{x})}$$



Use of this generating functional yields time reversed scattering processes

Opposite arrow of causality

Time reversal is anti-unitary

Lagrangian can be invariant, but PI is not

$$Z_+[J] \to Z_-[J]$$

Note: Also can be found in canonical quantization
Changes

$$[\phi(t,x), \pi(t,x')] = i\hbar\delta^3(x-x')$$

to

$$[\phi(t,x), \bar{\pi}(t,x')] = -i\hbar\delta^3(x-x') \quad \text{with} \quad \bar{\pi} = \frac{\partial \mathcal{L}}{\partial(\partial_\tau \phi)}.$$

“Arrow of time”:

Typical motivation:
"The laws of physics at the fundamental level don’t distinguish between the past and the future."

But this is not correct

The laws of quantum physics have an arrow of causality

Buried in the factors of i in the quantization procedures

Our time convention uses Z_+
- if reverse time convention used, use Z_-

Note: Arrow of thermodynamics follows arrow of causality

Merlin modes:

-Merlin (the wizard in the tales of King Arthur) ages backwards

"Now ordinary people are born forwards in Time, if you understand what I mean, and nearly everything in the world goes forward too. (...) But I unfortunately was born at the wrong end of time, and I have to live backwards from in front, while surrounded by a lot of people living forwards from behind."

T. H. White *Once and Future King*

Note, there is a key distinction with usual nomenclature "ghosts"

- ghost is anything with a minus sign in the numerator
- these Merlin modes refer to crucial sign $-i\gamma$ in denominator in addition

Dueling arrows of causality

Quartic propagators have opposing arrows

$$iD(q^2) \sim \frac{i}{q^2 + i\epsilon} \qquad \text{vs} \quad \sim \frac{-i}{q^2 - M^2 - i\gamma_M}$$

Who wins?

-massive state decays

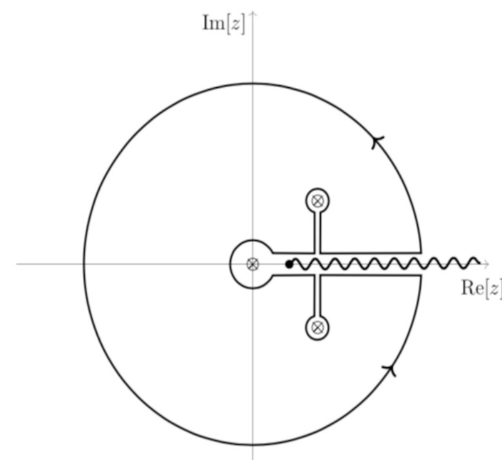
-stable states win

Revised Kallen-Lehmann representation

- due to Coleman (1969)

$$D(q) = \frac{1}{q^2 + i\epsilon} - \frac{\beta}{q^2 - M_r^2} - \frac{\beta^*}{q^2 - M_r^{*2}} + \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{q^2 - s + i\epsilon}$$

Three poles in this representation
- pair of complex conjugate poles
- spectral function quasi-Breit-Wigner



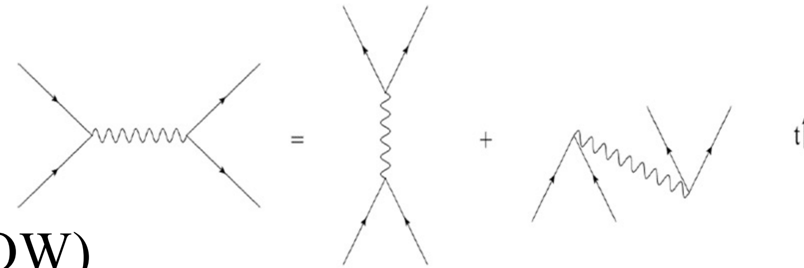
Old Lee-Wick literature neglects spectral pole
- crucial
- See Grinstein-O'Connell-Wise and D-M

Pathway to unitary predictions
- complex poles cancel imaginary parts (Anselmi's "fakeons")
- residual unitarity from spectral function

Phenomenology

Lee, Wick
Coleman
Grinstein, O'Connell, Wise
Alvarez, Da Roid, Schat, Szynkman

Vertex displacements: (ADSS)
- look for final state emergence
- before beam collision

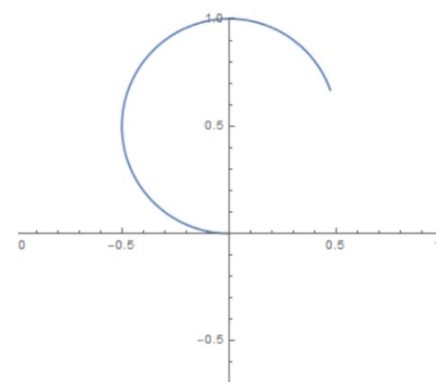


Form wavepackets – early arrival (LW, GOW)
- wavepacket description of scattering process
- some components arrive at detector early

Resonance Wigner time delay reversal
- normal resonaces counterclockwise on Argand diagram

$$\Delta t \sim \frac{\partial \delta}{\partial E} \sim> 0$$

- Merlin modes are clockwise resonance



For gravity, all are Planck scale
- no conflict with experiment

Living with Causality Uncertainty

Wavepackets are an idealization:
-really formed by previous interactions

Likewise beam construction from previous scattering
- and measurement due to final scattering

The timing of scattering will become **uncertain**

But causal uncertainty is likely a general property of quantum gravity

Unitarity of unstable particles:

$$\langle f|T|i\rangle - \langle f|T^\dagger|i\rangle = i\sum_j \langle f|T^\dagger|j\rangle\langle j|T|i\rangle$$

Who counts in unitarity relation?

- Veltman 1963
- **only stable particles count**
- they form asymptotic Hilbert space
- **do not** make any cuts on unstable resonances

UNITARITY AND CAUSALITY IN A RENORMALIZABLE FIELD THEORY WITH UNSTABLE PARTICLES

M. VELTMAN *)

This looks funny from free-field quantization

- interaction removes states from the Hilbert space

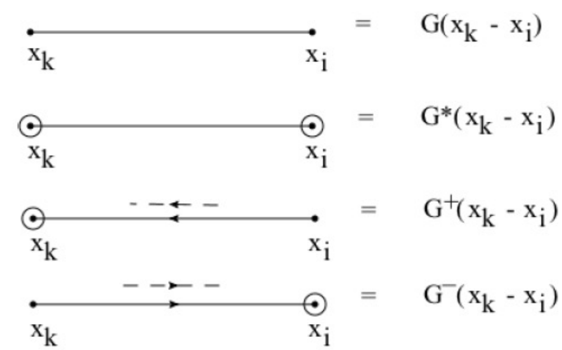
Also, we know some states are almost stable

- can treat them as essentially stable
- Narrow Width Approximation (NWA)

But of course, Veltman is correct

Formal proof of unitarity with unstable ghosts

With G. Menezes arXiv:1908.02416 in PRD



Follows Veltman:
- circling rules
- largest time equation
- turns into derivation of cutting rules

Only difference is **energy flow**

$$-iG^*(x-x') \;=\; \Theta(x_0-x_0')G^-(x-x')+\Theta(-x_0+x_0')G^+(x-x')$$
$$-i\widetilde{G}^*(x-x') \;=\; \Theta(x_0-x_0')\widetilde{G}^-(x-x')+\Theta(-x_0+x_0')\widetilde{G}^+(x-x')$$
$$-i\widetilde{G}^*_{\rm GH}(x-x') \;=\; \Theta(x_0-x_0')\widetilde{G}^+_{\rm GH}(x-x')+\Theta(-x_0+x_0')\widetilde{G}^-_{\rm GH}(x-x')$$

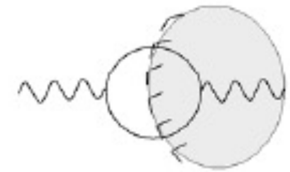
Important point - all steps in Minkoswki space
- no analytic continuation employed

Formalizes early work by Lee-Wick

Unitarity: Cutkosky cutting rules

Obtain discontinuity by replacing propagator with:

$$\frac{i}{q^2 - m^2 + i\epsilon} \to 2\pi\delta(q^2 - m^2)\theta(q_0)$$



Also on far side of cut, use:

$$\frac{i}{q^2 - m^2 + i\epsilon} \to \frac{-i}{q^2 - m^2 - i\epsilon}$$

Example – self energy

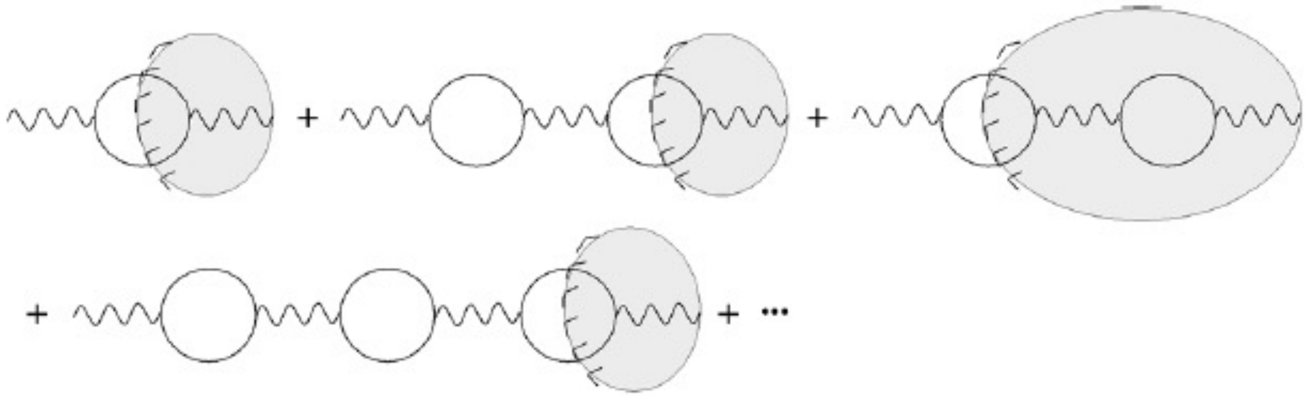
$$\mathrm{Disc}_2\ \Sigma(q) = \frac{\kappa^2 q^4 (N+1)}{2} \int \frac{d^4k}{(2\pi)^4}\ 2\pi\delta(k^2)\theta(k_0)\ 2\pi\delta((q-k)^2)\theta((q-k)_0)$$

Can repackage this:

$$\mathrm{Disc}_2\ \Sigma(q) = 2q\Gamma_2(q)$$

The discontinuity is equivalent to the decay width at q^2

<u>Cuts in a resonance propagator</u>:

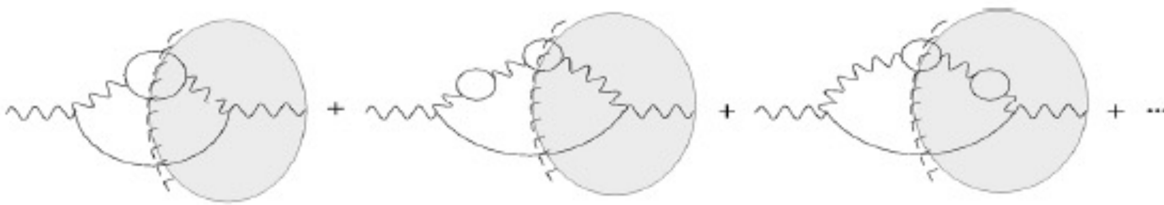


Bubble sum on each side of propagator:
- will c.c. propagators on the far side

$$\text{Disc } D(q) = D(q)\ 2q\Gamma_2(q)\ D^*(q) = -2\ \text{Im}[D(q)]$$

This is true no matter if normal resonance or Merlin modes
- imaginary parts are the same

Three particle cut = resonance + stable cut



$$\mathrm{Disc}_3\,\Sigma(q) = \kappa^2 q^4 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{1}{(q-k_1)^2-\frac{\kappa^2(q-k_1)^4}{2\xi^2}}\frac{(N+1)\kappa^2(q-k_1)^2}{2}$$
$$\times\;2\pi\delta(k_1^2)\theta(k_{10})\;2\pi\delta(k_2^2)\theta(k_{20})\;2\pi\delta((q-k_1-k_2)^2)\theta((q-k_1-k_2)_0)\frac{1}{(q-k_1)^2-\frac{\kappa^2(q-k_1)^4}{2\xi^2}}$$

Identify matrix element

$$\mathcal{M}_3 = \kappa q^2 \kappa (q-k_1)^2 D(q-k_1)$$

and play similar games, to get expected unitarity relation

$$\mathrm{Disc}_3\,\Sigma(q) = 2q\Gamma_3(q)$$

Again result is independent of type of resonance

Bottom line: discontinuities come from cuts on stable particles

Narrow width approximation

Discontinuity in propagator was due to on-shell states only

$$\text{Disc } D(q) = D(q)\ 2q\Gamma(q)\ D^*(q) = \frac{2q\Gamma(q)}{(q^2 - m_r^2)^2 + (m_r\Gamma(q))^2}$$

But when Γ is small, this is highly peaked on the resonance,
Use:

$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$$

Limits to usual cutting rule:

$$\lim_{\Gamma \to 0} \text{Disc } D(q) = 2\pi\delta(q^2 - m_r^2)$$

In "three particle cut" , this is equiv. decay to resonance plus stable

Again, this result is independent of normal or Merlin resonance

Lessons ala Veltman

Physics:
Cuts for resonances actually are through the stable particles

Resonances do not go on-shell

Math:
The $i\gamma$ quickly overwhelms the $i\epsilon$

In the end, this is what Veltman 1963 shows

Think: LSZ

$$\langle b|S|a\rangle = \left[i\int d^4x_1 e^{-ip_1\cdot x_1}(\Box_1 + m^2)\right]\cdots\left[i\int d^4x_n e^{ip_n\cdot x_n}(\Box_n + m^2)\right]\langle\Omega|T\{\phi(x_1)\cdots\phi(x_n)\}|\Omega\rangle$$

Heuristic proof of unitarity

Unitarity works with stable particle as external states

Cuts through stable particle loops same for normal and Merlin resonances

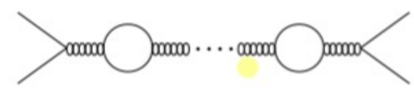
Both normal states and Merlin resonances can be in same propagator

Veltman proved normal resonances satisfy unitarity to all orders

The Merlins will then also satisfy unitarity

Unitarity in the spin two channel

Do these features cause trouble in scattering?
- consider scattering in spin 2 channel



First consider single scalar at low energy:

$$i\mathcal{M} = \left(\frac{1}{2}V_{\mu\nu}(q)\right)\left[iD^{\mu\nu\alpha\beta}(q^2)\right]\left(\frac{1}{2}V_{\alpha\beta}(-q)\right)$$

$$\mathcal{M} = 16\pi \sum_{J=0}^{\infty}(2J+1)T_J(s)P_J(\cos\theta)$$

Results in

$$T_2(s) = -\frac{N_{\text{eff}}s}{640\pi}\bar{D}(s).$$

$N_{\text{eff}} = 1/6$ for a single scalar field

$$\bar{D}^{-1}(s) = \frac{1}{\tilde{\kappa}^2}\left\{1 - \frac{\tilde{\kappa}^2 s}{2\xi^2(\mu)} - \frac{\tilde{\kappa}^2 s N_{\text{eff}}}{640\pi^2}\ln\left(\frac{s}{\mu^2}\right) + \frac{i\tilde{\kappa}^2 s N_{\text{eff}}}{640\pi}\right\}$$

Satisfies elastic unitarity:

$$\mathrm{Im}T_2 = |T_2|^2.$$

This implies the structure

$$T_2(s) = \frac{A(s)}{f(s) - iA(s)} = \frac{A(s)[f(s) + iA(s)]}{f^2(s) + A^2(s)}$$

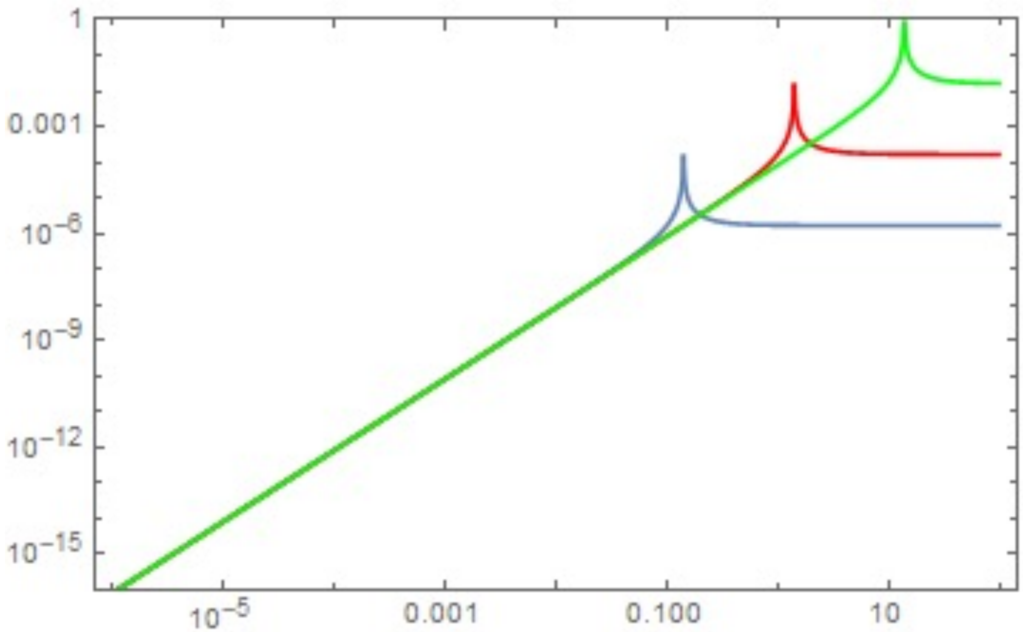
for any real *f(s)*

Signs and magnitudes work out for $A(s) = -\frac{N_{\text{eff}} s}{640\pi}.$

Multi-particle problem:

- just diagonalize the J=2 channel
- same result but with general N

Scattering amplitude at weak coupling:



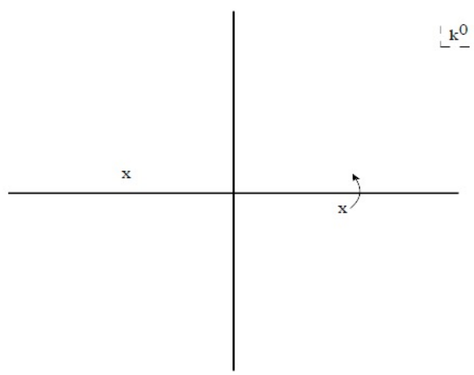
$\xi^2 = 0.1, 1, 10$

Narrow Width Approximation with Merlin modes

This path follows M. Schwartz: QFT +SM

$$i\mathcal{M} = -\int \frac{d^4k}{(2\pi)^4} \frac{i}{(k-q)^2 + i\epsilon} \frac{-i}{k^2 - m^2 - i\gamma}$$

Normal



Convert D_F to advanced propagator

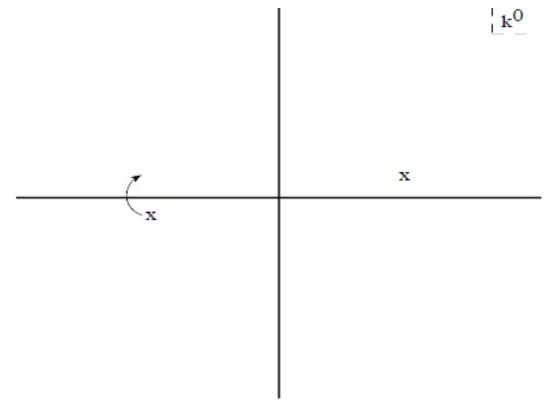
$$\frac{i}{k^2 + i\epsilon} = iD_A(k) + \frac{\pi}{\omega_k}\delta(k_0 - \omega_k)$$

Product of advanced propagators vanishes

Play some games and pick out Im part

$$\text{Im}[\mathcal{M}] = -\int \frac{d^4k}{(2\pi)^4} \left[\pi\delta((k-q)^2)\ \frac{\pi}{E_k}\delta(k_0 - E_k) - \frac{\pi}{\omega_k}\delta(k_0 - q_0 - \omega_k)\ \pi\delta(k^2 - m^2)\right]$$

Ghost



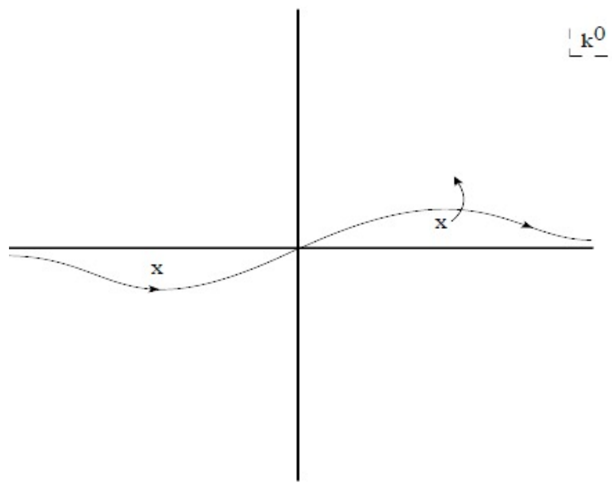
Take same path

$$\frac{-i}{k^2 - m^2 - i\gamma} = -iD_{``A"}(k) + \frac{\pi}{E_k}\delta(k_0 + E_{k-p})$$

But delta functions cannot be satisfied

$$\text{Im}[\mathcal{M}] = -\int \frac{d^4k}{(2\pi)^4} \left[\pi\delta((k-q)^2)\ \frac{\pi}{E_k}\delta(k_0 + E_k) - \frac{\pi}{\omega_k}\delta(k_0 - q_0 - \omega_k)\ \pi\delta(k^2 - m^2)\right]$$

Lee-Wick contour



Contour goes around poles
Returns to normal discontinuity as γ goes to zero

Compatible with usual Wick rotation

Treating ghost like a normal particle requires LW contour

The theory is still not fully understood:

Stability at higher curvatures/energies

- closer to high mass pole
- if unstable there, is it benign? (like Starobinsky inflation)

More detailed explicit calculations

- Gabriel has one not yet published
- higher order loops
- generalization of Lee-Wick contour **(Is it possible at higher loops?)**

Connection to unitarity-based calculations (Menezes 2021)

-unitarity techniques with unstable particles

Lattice simulations?

- but Euclidean vs. Lorentzian

Etc…

Summary:

Quadratic gravity is a renormalizeable quantum field theory

Positive features:

- massless graviton identified through pole in propagator
- ghost resonance decays – does not appear in spectrum
- seems stable under perturbations near Minkowski
- unitarity with only stable asymptotic states
- LW contour as shortcut via narrow width approximation

Most unusual feature:

- causality violation/uncertainty near Planck scale

More work needed, but so far appears as a viable option for QG